

REVISITING NUCLEOSYNTHESIS CONSTRAINTS ON PRIMORDIAL MAGNETIC FIELDS

Dario Grasso[†] and Hector R. Rubinstein

Department of Theoretical Physics, Uppsala University

Box 803 S-751 08 Uppsala, Sweden, and

Department of Physics, University of Stockholm

Vanadisvägen 9, S-113 46 Stockholm, Sweden.

Abstract

In view of several conflicting results, we reanalyze the effects of magnetic fields on the primordial nucleosynthesis. In the case the magnetic field is homogeneous over a horizon volume, we show that the main effects of the magnetic field are given by the contribution of its energy density to the Universe expansion rate and the effect of the field on the electrons quantum statistics. Although, in order to get an upper limit on the field strength, the weight of the former effect is numerically larger, the latter cannot be neglected. Including both effects in the PN code we get the upper limit $B \leq 1 \times 10^{11}$ Gauss at the temperature $T = 10^9$ $^{\circ}K$. We generalize the considerations to cases when instead the magnetic is inhomogeneous on the horizon length. We show that in these cases only the effect of the magnetic field on the electrons statistics is relevant. If the coherence length of the magnetic field at the end of the PN is in the range $10 \ll L_0 \ll 10^{11}$ cm our upper limit is $B \leq 1 \times 10^{12}$ Gauss.

Keywords: Primordial nucleosynthesis, primordial magnetic fields.

PACS numbers: 98.80.Cq; 98.62.En.

[†] EEC Twinning Fellow

The study of the effects of magnetic fields on the primordial nucleosynthesis (PN) began with the pioneering works of Greenstein [1] and Matese and O'Connell [2]. In a previous paper Matese and O'Connell [3] first observed that magnetic fields larger than the critical value $B_c \equiv eB/m_e^2 = 4.4 \times 10^{13}$ Gauss can affect the β -decay rate, mainly through the modification of the phase space of the electrons and positrons. Greenstein advanced that, if magnetic fields were present in the early Universe, then this can affect not only the weak processes regulating the neutron to proton ratio in the early Universe, namely

$$\begin{aligned} n + e^+ &\rightleftharpoons p + \bar{\nu} \\ n + \nu &\rightleftharpoons p + e^- \\ n &\rightleftharpoons p + e^- + \bar{\nu}, \end{aligned}$$

but also the expansion rate of the Universe. These effects will have consequences on the PN. However, they are competing effects. In fact, whereas the magnetic field energy density $\rho_B = B^2/8\pi$ accelerates the Universe expansion and therefore increases the predicted abundance of the relic ${}^4\text{He}$, the effect of the field on the weak processes is to increase the $n \rightarrow p$ conversion rate reducing the predicted ${}^4\text{He}$ abundance. Greenstein argued that the former effect dominates over the latter. In a following paper, Matese and O'Connell [2] improved Greenstein's considerations by computing explicitly $n \rightarrow p$ conversion rate at the PN time in presence of a strong magnetic field and they confirmed Greenstein's qualitative conclusions. The comparison of the PN predictions for the relic light isotope abundances with the observations provides a criteria to bound the magnetic field strength at the nucleosynthesis time [4]. However, Matese and O'Connell did not provide any upper limit.

Recently, the interest about magnetic fields in the early Universe received new impulse from the idea that these fields might be produced during the electroweak phase transition[5] or inflation [6]. The renewed interest in this subject led several authors to investigate Matese and O'Connell's treatment again.

Cheng, Schramm and Truran [7] first used the standard nucleosynthesis code [8] to get an upper limit on the magnetic field strength. The effects of the field they considered are the same studied by Matese and O'Connell. However Cheng et al. disagree with Matese and O'Connell in their conclusions. In fact, they claim that the effect of the magnetic field on the weak reactions is the most important. Cheng at al. obtained the following upper limit: $B < 10^{11}$ Gauss at the end of the nucleosynthesis ($T = 10^9$ $^{\circ}\text{K}$). In both the Matese and O'Connell and Cheng at al. papers the magnetic field was assumed to be uniform over an horizon volume. We will return below to the crucial implications of this assumption.

Recently, we reconsidered this subject [9]. Besides the effects of the magnetic field on the reaction rates, we considered the effect of the field on the statistical distribution of the electrons and positrons. In fact, due to the effect of the magnetic field on the electron wave function, the phase space of the lowest Landau

level is enhanced. If the temperature is small with respect to the energy gap between the lowest and first excited level, that is if $eB \ll T^2$ ¹, a relevant fraction of electron-positron pairs will *condense* in the lowest Landau level affecting the statistical distribution of the electron gas. As a consequence, the number density and energy density of electrons and positrons are increased with respect to the case where the magnetic field is not present. The expressions for the electron+positron energy density and pressure are

$$\rho_e = \frac{\gamma}{2\pi^2} m_e^4 \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int_{\sqrt{1+2(n+1)\gamma}}^{\infty} d\epsilon \frac{\epsilon^2}{\sqrt{\epsilon^2 - 1 - 2(n+1)\gamma}} \quad (1)$$

$$\left(\frac{1}{1 + e^{\frac{m_e \epsilon}{T} + \mu}} + \frac{1}{1 + e^{\frac{m_e \epsilon}{T} - \mu}} \right)$$

$$p_e = \frac{\gamma}{6\pi^2} m_e^4 \sum_{n=0}^{\infty} (2 - \delta_{n0}) \int_{\sqrt{1+2(n+1)\gamma}}^{\infty} d\epsilon \frac{(\epsilon^2 - 1)}{\sqrt{\epsilon^2 - 1 - 2(n+1)\gamma}} \quad (2)$$

$$\left(\frac{1}{1 + e^{\frac{m_e \epsilon}{T} + \mu}} + \frac{1}{1 + e^{\frac{m_e \epsilon}{T} - \mu}} \right) .$$

Here $\gamma \equiv B/B_c$, ϵ and μ are the electron, or positron, energy and chemical potential in mass units, and n labels the Landau level.

It is evident that the modified contribution of electrons and positrons to the total energy density will affect the expansion rate of the Universe. The entropy content of the Universe will be also changed by the effect on the field on the electrons thermodynamics. However, for values of the ratio $eB/T^2 \sim 1$, the main effect of the modified electron statistics on PN is on the variation that it induces on the time derivative of the photon temperature. This derivative is given by

$$\frac{dT}{dt} = -3H \frac{\rho_{em} + p_{em}}{d\rho_{em}/dT} \quad (3)$$

where $\rho_{em} = \rho_e + \rho_\gamma$, $p_{em} = p_e + p_\gamma$, H is the Universe expansion rate and T is the photon temperature. For small values of the ratio eB/T^2 , the most relevant effect of the magnetic field enters in the derivative $d\rho_{em}/dT$, that is smaller than the value it would have if the field were not present. More physically, this effect can be interpreted as a delay in the electron-positron annihilation time induced by the magnetic field. This will give rise to a slower entropy transfer from the electron-positron pairs to the photons, then to a slower reheating of the heat bath. In fact, due to the enlarged phase-space of the lowest Landau level of electrons and positrons, the equilibrium of the process $e^+e^- \leftrightarrow \gamma$ is shifted towards its left side. As we already showed in ref.[9], and we are going to discuss in more details here, this effect has a clear signature on the Deuterium and ^3He predicted abundances besides that on the ^4He abundance.

¹Observe that, neglecting dissipation, eB/T^2 remains constant during the Universe expansion.

Including the effect of the magnetic field on the electron statistics, together with the effect of the field energy density and the effect on the weak processes in the standard nucleosynthesis code ² we got the upper limit $B \leq 3 \times 10^{10}$ Gauss. We also considered other kind of effects like that of the magnetic field on the nucleon [10] and electron masses, but these effects proved to be subdominant.

Recently our conclusions have been put in question in a preprint by Kernan, Starkman and Vachaspati [11]. In this preprint, the effect of the magnetic field on the PN has been parameterized in terms of an effective number of neutrinos in order to avoid the numerical PN computations. Kernan et al. considered, at least in principle, all the effects of the magnetic field that we discussed above, including that on the electron statistics. However, although for opposite reasons, their main conclusion disagree both with Cheng et al. and our results. In fact, in Kernan et al. opinion the most relevant consequence of the magnetic field on the PN is the one mediated by the effect of the field energy density on the expansion rate of the Universe. The upper limit on the strength of the magnetic field they got is $eB/T_\nu^2 \leq 1$ that can be read $B \leq 1 \times 10^{11}$ at $T = 10^9$ °K. This result does not differ from Cheng at al. [7] numerical result although it does in its physical interpretation.

The aim of this letter is to try to clarify this confused situation. As we are going to show, some mistakes are probably present in all refs.[7][11][9]. To start with, we have recently realized that our upper limit, in ref.[9] is incorrect. This mistake was due to the use of a wrong version of the standard nucleosynthesis code. We have now rerun the right version of the PN code [12] with an improved subroutine that evaluates the effect of the magnetic field on the electron and positron energy density, pressure and energy density time derivative. We considered also the effects of the magnetic field on the weak reaction and the effect of the field energy density on the Universe expansion rate. Including all possible effects in the PN code we get the predictions we report in table I.

These results have been obtained using $N_\nu = 3$, the lowest value of the neutron half life compatible with experimental data, ($\tau_{1/2}(n) = 885$ s.) and the smallest value of the baryons to photons ratio that makes the predicted $(D+{}^3\text{He})/\text{H}$ ratio, in absence of the magnetic field, compatible with the astrophysical observations ($\eta = 2.8 \times 10^{-10}$). This choice assured the minimal predicted abundance of the ${}^4\text{He}$ for each value of the magnetic field strength that we considered. Requiring that such abundance do not exceed the value of 0.245 [13] we infer from our data the upper limit

$$B \leq 1 \times 10^{11} \text{ Gauss} \quad (4)$$

when the temperature was $T = 10^9$ °K (end of PN).

In order to discriminate the relative weight of the effects we considered we have run our code leaving on only one effect at a time. We start considering only the effect of the magnetic field energy density. As expected, we see in Tab. II that

²However we used here an old version of the code, see [8].

Table I. Here we report the PN predictions for the relic light element abundance for several values of the parameter $\gamma \equiv B/B_C$ given at the temperature $T = 10^9$ °K.

$\gamma(T = 10^9$ °K)	${}^4\text{He}$	$(\text{D} + {}^3\text{He})/\text{H}$	${}^7\text{Li}/\text{H}$
0	0.237	1.04×10^{-4}	1.15×10^{-10}
1×10^{-3}	0.240	1.02×10^{-4}	1.17×10^{-10}
2×10^{-3}	0.242	1.02×10^{-4}	1.19×10^{-10}
3×10^{-3}	0.244	1.03×10^{-4}	1.19×10^{-10}
4×10^{-3}	0.246	1.05×10^{-4}	1.20×10^{-10}
5×10^{-3}	0.249	1.08×10^{-4}	1.21×10^{-10}
6×10^{-3}	0.252	1.12×10^{-4}	1.22×10^{-10}
7×10^{-3}	0.255	1.16×10^{-4}	1.24×10^{-10}
8×10^{-3}	0.258	1.22×10^{-4}	1.27×10^{-10}
9×10^{-3}	0.262	1.28×10^{-4}	1.31×10^{-10}
1×10^{-2}	0.266	1.35×10^{-4}	1.36×10^{-10}

the predicted ${}^4\text{He}$ abundance grows with the field strength. In fact, larger values of B induce a larger Universe expansion rate then a higher freeze-out temperature for the n/p ratio.

In Tab. III we report instead our predictions in the case only the effect of the magnetic field on the electron quantum statistic is considered. Note that, in order to display the relative weight of the effects that we considered, deriving the results of Tab. II and III we kept the values of $\tau_{1/2}(n)$ and η fixed and equal to the value we used for Tab. I.

It is interesting to observe the behavior of the sum of the predicted abundances for the Deuterium and ${}^3\text{He}$ while varying the magnetic field strength. This is qualitatively different from what reported in Tab. II. In fact, if only the effect on the electron statistics is considered, the $(\text{D} + {}^3\text{He})/\text{H}$ abundances ratio decreases when the field strength increases (see also Fig.2). At the same time, the predicted ${}^4\text{He}$ increases increasing B (see Fig.1).

From the results we reported in Tab. II it follows that the upper limit we would get if only the effect of the magnetic field energy density were present is $B \leq 2 \times 10^{11}$ Gauss whereas considering only the effect on the electrons quantum statistics (see Tab. III) we would get $B \leq 5 \times 10^{11}$ ³. Thus, contrary to our previous claim [9], the former effect numerically dominates the latter. However, from Fig. 1 it is clear at a glance that the effect of the magnetic field on the electron statistics cannot be neglected and indeed it dominates for small values

³We have to point out that this limit has been obtained using only our predictions for the ${}^4\text{He}$ abundance keeping the value of η fixed. However, since η is an unknown parameter, it should be rescaled for every chosen value of B in such a way to get the largest $\text{D} + {}^3\text{He}$ abundance compatible with the observational data (see below).

Table II. Here we report the PN predictions obtained considering only the effect of the magnetic field energy density on the Universe expansion.

$\gamma(T = 10^9 \text{ }^{\circ}\text{K})$	${}^4\text{He}$	$(\text{D}+{}^3\text{He})/\text{H}$	${}^7\text{Li}/\text{H}$
0	0.237	1.04×10^{-4}	1.15×10^{-10}
1×10^{-3}	0.237	1.05×10^{-4}	1.15×10^{-10}
2×10^{-3}	0.238	1.06×10^{-4}	1.15×10^{-10}
3×10^{-3}	0.240	1.09×10^{-4}	1.15×10^{-10}
4×10^{-3}	0.241	1.12×10^{-4}	1.16×10^{-10}
5×10^{-3}	0.244	1.17×10^{-4}	1.17×10^{-10}
6×10^{-3}	0.246	1.22×10^{-4}	1.19×10^{-10}
7×10^{-3}	0.249	1.28×10^{-4}	1.23×10^{-10}
8×10^{-3}	0.252	1.36×10^{-4}	1.28×10^{-10}
9×10^{-3}	0.256	1.44×10^{-4}	1.34×10^{-10}
1×10^{-2}	0.259	1.53×10^{-4}	1.42×10^{-10}

of the field strength. In this sense we still disagree with the conclusion of the authors of ref.[11]. It may be that in ref.[11] the effect of the magnetic field on the electrons and positrons statistical distribution was not treated properly close to the saturation of the upper limit on B , that is for $eB/T^2 \sim 1$.

Concerning the effect of the magnetic field on the weak reactions we have verified that such effect is negligible with respect to the others. Mass changes are also negligible in the range of field strengths that we considered. About these conclusions we agree with the authors of the refs.[2] and [11]. Since the numerical predictions of ref.[7] do not differ significantly from those of ref.[11] and the results we reported in Tab. II we think that the different physical interpretation of the results given in ref.[7] is probably due to an oversight writing that paper.

To summarize, we have found that the main effects of a magnetic field on the PN plays through the action of the field on the electrons and positrons quantum statistics and the direct effect of the field energy density on the expansion rate of the Universe. Although the latter effect is numerically larger than the former, these effects are indeed comparable, at least if the magnetic field is uniform over the horizon scale as we assumed so far.

Let us now discuss how our previous consideration will be affected if we relax the uniformity assumption. Before discussing more model dependent motivations to consider inhomogeneous magnetic fields, it is mandatory to compare the upper limit we got above with the upper limit one would get considering the effect of an homogeneous cosmic magnetic field on the early Universe isotropy. In fact, since magnetic fields break rotational invariance they can induce an anisotropy in the Universe expansion. This anisotropy can have observable consequences on the isotropy of the cosmic background radiation and on the PN [14][15]. Using

Table III. Here we report the PN predictions obtained considering only the effect of the magnetic field energy density on the electrons and positrons quantum statistics.

$\gamma(T = 10^9 \text{ }^{\circ}\text{K})$	${}^4\text{He}$	$(\text{D}+{}^3\text{He})/\text{H}$	${}^7\text{Li}/\text{H}$
0	0.237	1.04×10^{-4}	1.15×10^{-10}
1×10^{-3}	0.240	1.01×10^{-4}	1.17×10^{-10}
2×10^{-3}	0.241	9.99×10^{-5}	1.19×10^{-10}
3×10^{-3}	0.241	9.89×10^{-5}	1.20×10^{-10}
4×10^{-3}	0.242	9.81×10^{-5}	1.21×10^{-10}
5×10^{-3}	0.243	9.72×10^{-5}	1.22×10^{-10}
6×10^{-3}	0.243	9.65×10^{-5}	1.22×10^{-10}
7×10^{-3}	0.243	9.59×10^{-5}	1.23×10^{-10}
8×10^{-3}	0.244	9.51×10^{-5}	1.24×10^{-10}
9×10^{-3}	0.244	9.46×10^{-5}	1.24×10^{-10}
1×10^{-2}	0.244	9.40×10^{-5}	1.25×10^{-10}
1.1×10^{-2}	0.245	9.35×10^{-5}	1.26×10^{-10}
1.2×10^{-2}	0.245	9.28×10^{-5}	1.26×10^{-10}

these criteria Zeldovich and Novikov [14] got the limit $B \leq 4 \times 10^{10}$ Gauss for a magnetic field homogeneous at least over a horizon volume considered at the temperature $T = 10^9$ K. This limit is roughly one order of magnitude more stringent than the one we got above as those obtained in refs.[7] and [11] as well.

Nevertheless, if the magnetic field is inhomogeneous with a coherence length much smaller than the Hubble radius we do not have to care about the Zeldovich and Novikov upper limit. In our opinion this is the most plausible physical scenario. It is in agreement with the predictions of most of the models for magnetic field generation in the early Universe [16]. Among these models we are mainly interested in those that predict that magnetic fields are generated during the electroweak phase transition. The predicted coherence length of the field at the electroweak phase transition time is $L_0 \approx m_W^{-1} \sim 10^{-16} H_{ew}^{-1}$ (where H_{ew}^{-1} is the Hubble radius at that time) in the case the transition is second order [5] or, perhaps a more probable scenario, $L_0 \sim R_{bubble} \approx 10^{-(2 \div 3)} H_{ew}^{-1}$ for a first order phase transition [17][18]. Assuming the magnetic field strength decreases only due the Universe expansion the ratio L_0/H^{-1} remains constant in time. In both cases it is evident that $L_0 \ll H^{-1}$.

Having magnetic fields that fluctuate in space on a typical scale $L_0 \ll H^{-1}$ forces us to reconsider the analysis we made in the first part of this letter paying some attention to the different scales on which the effects of the field on the PN act and the nature of the averaging processes.

To start with, let consider the effect of the field on the weak processes. The mean free path between two weak reactions is $\Gamma_W \sim G_F^2 T^5$ that is of the order

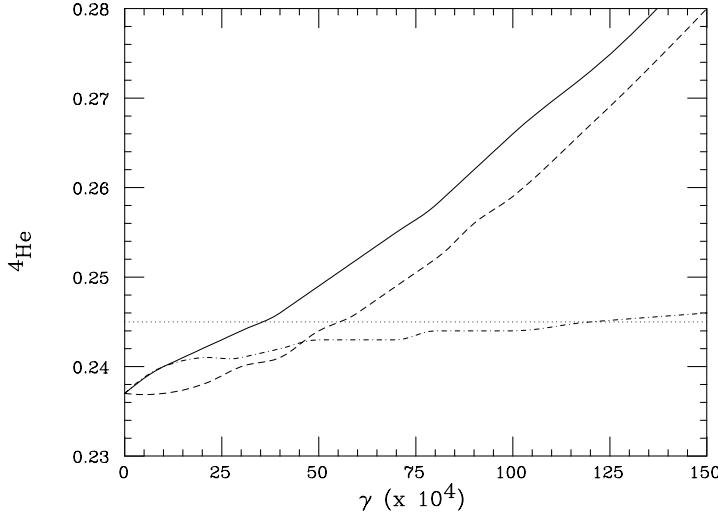


Figure 1: The ${}^4\text{He}$ predicted abundance is represented in function of the parameter γ , considered at $T = 10^9 \text{ }^{\circ}\text{K}$, in three different cases: only the effect of the magnetic field energy density is considered (dashed line); only the effect of the field on the electron statistics is considered (dotted-dashed line); both effects are considered (continuous line). The dotted line represents the observational upper limit.

of the horizon radius at the onset of the PN ($H^{-1}(T \sim 10^{10} \text{ }^{\circ}\text{K}) \sim 10^{10} \text{ cm}$). It is clear that if $L_0 \ll \Gamma_w^{-1}$ the charged particles involved in the process will feel a magnetic field changing a number of times between the two reactions so that the effect of the field will be almost averaged out [19]. At any rate, the effect of the field on the weak reaction rates was already subdominant in the case B were assumed to be homogeneous.

More relevant is, instead, the effect of the inhomogeneity of the magnetic field on the action that the magnetic field energy density ρ_B plays on the Universe expansion. As discussed in detail in refs.[19][20], since in this case the field is fluctuating in space, only a *mean* magnetic field within a horizon volume can be defined as a meaningful quantity. This quantity is defined by

$$\bar{\rho}_B = \frac{1}{2V_H} \int_{L_0}^{L_H} d^3r B_{rms}^2(r) \quad (5)$$

where

$$B_{rms} = B_0 \left(\frac{T}{T_0} \right)^2 \left(\frac{L_0}{L} \right)^p \quad (6)$$

is the root-mean-squared field strength. Here p is a parameter that depends on the statistical configuration of the magnetic field. Typical values of p are

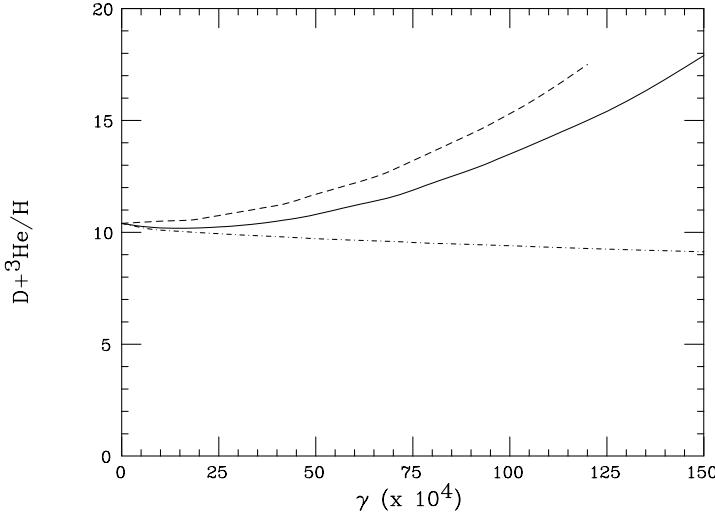


Figure 2: The $(D+^3He)/H$ predicted abundance is represented for the same cases illustrated in Fig.1.

$1/2$, 1 , $3/2$. Inserting Eq. (6) in Eq. (5) it is evident that $\bar{\rho}_B$ will suffer a huge suppression if $L_0 \ll L_H$ so that it will have no chances to affect any more the PN. The impact of the magnetic field on the electron quantum statistics is much less affected than the others by the inhomogeneity of the field. In fact, in this case, the characteristic length scale we have to compare to L_0 is the Compton scattering length of the electrons λ_C , since this scattering is responsible for the electron thermalization, and the radius of periodic motion of the electrons in the plane normal to the magnetic field vector. At the PN temperature ($T \sim 10^{9 \div 10} \text{ } ^\circ K$) the Compton cross section does not differ significantly from the Thomson cross section. Then we have

$$\lambda_C = (\sigma_T n_e v)^{-1} \sim 10^{-(2 \div 5)} \text{ cm.} \quad (7)$$

This has to be compared with the Hubble radius at the same temperature that is $\sim 10^{10 \div 11}$ cm. The radius of the classical orbit of the electron in an over critical magnetic fields is given by

$$R_n = \left(\frac{n+1}{\gamma} \right)^{1/2} m_e^{-1} \quad (8)$$

where n labels the Landau level. Since the magnetic field plays its main effect on the lowest Landau level it is clear that the radius we have to care about is much smaller than λ_C and far below the expected value of L_0 at the PN time.

Since, $\lambda_C \ll \Gamma_W^{-1} \ll H^{-1}$ we see that the effect of the magnetic field on the electron quantum statistics provides the best probe of the field at small length scales. If $\lambda_C \ll L_0 \ll H^{-1}$ this is the only effect we remain with.

The PN predictions of Tab. III cannot be used in their present form to determine, in this case, the upper limit on the magnetic field strength. In fact, as we pointed-out above, obtaining Tab. III we kept the value of η fixed. However, η is an unknown parameter and its value has to be bound from below in order the predicted abundances of D and ^3He do not exceed the relic abundances extrapolated from present observations. Since in our case the predicted of abundances of these isotopes depend on the field strength, we have to rescale η for every value of B in order to get their largest predicted abundances compatible with the observational upper limit. According to ref. [13] we assume this limit to be $(\text{D} + ^3\text{He})/\text{H} \leq 1.1 \times 10^{-4}$. Finally, we have to compare the predicted abundance of the ^4He , obtained using such value of η , with the maximal abundance compatible with observations $^4\text{He} \leq 0.245$ [13]. Using these procedure we get the upper limit

$$B \leq 1 \times 10^{12} \text{ Gauss.} \quad (9)$$

Again, this limit refers to the temperature $T = 10^9 \text{ }^{\circ}\text{K}$. The corresponding value of η is 2.3. As expected, this value is smaller than the standard lower limit $\eta \geq 2.8$, since the effect of the magnetic field on the electron statistics suppresses the predicted abundance of D and ^3He . It is worthwhile to note here that, although this effect increases also the predicted abundance of ^4He , large magnetic fields might help to alleviate the “nucleosynthesis crisis” that is presently under debate [21].

So far we neglected any dissipative effect in the plasma. However, even in a relativistic plasma conductivity σ is a finite quantity. This means that any field configuration on a length scale $L_0 \ll L_{diss}$, where

$$L_{diss} = \sqrt{\frac{H^{-1}}{4\pi\sigma}}, \quad (10)$$

will be dissipated. It is interesting to observe that if $eB/T^2 \gg 1$ the conductivity properties of the plasma will be seriously affected by the magnetic field. In fact, in this case the e^+e^- pair number density is increased by the effect of the field on the electrons and positrons phase-space. As a consequence this will make the plasma conductivity to decrease then L_{diss} to grow [22]. This can have important consequences on the field evolution after it is generated in the early Universe. However, such effect is negligible at the PN time. Following ref. [20] L_{diss} at $T = 10^9 \text{ }^{\circ}\text{K}$ MeV can be estimated to be

$$L_{diss} \simeq 0.1 g_*^{-1/4} \left(\frac{10^9}{T} \right)^{7/4} \approx 1 \div 10 \text{ cm.} \quad (11)$$

Taking this considerations into account we conclude that our upper limit (9) applies only if $10 \ll L_0 \ll 10^{11}$ cm.

It is worthwhile to see how our upper limit constraints at least one of the recent models of relic magnetic field generation. Assuming the field is generated at the end of a first order electroweak phase-transition [17][18], within large theoretical uncertainties the predicted magnetic field strength at the temperature $T \sim 10^{15}$ $^{\circ}$ K is $\approx 10^{24}$ Gauss with a coherence length $L_0 \approx 10^{-(2 \div 3)}$ cm. If the magnetic field flux above this length scale remains frozen in the plasma, so that the field strength decrease only due to the Universe expansion, at the PN time we expect: $B(T = 10^9$ $^{\circ}$ K) $\sim 10^{12}$ Gauss and $L_0 \sim 10^{4 \div 5}$ cm. Thus, this model is not excluded by PN considerations.

In conclusion, in this letter we unfortunaly revise upwards previous limits on homogeneous magnetic field strengths at PN time by a factor 10. Earlier calculations contained mistakes on a variety of points as discussed. As a consequence, previous bounds based on Universe isotropy remain more stringent[14]. However, in the case the magnetic field is inhomogeneous on the horizon length scale, isotropy considerations do not apply. In this case, we showed that the main observable effect of the magnetic field on the PN is on the electron and positron quantum statistics.

Acknowledgments

The authors wish to thank V. Semikoz for valuable discussions. This work was supported in part by the Swedish Research Council and a Twinning EEC contract.

References

- [1] G. Greenstein, *Nature* **233**, (1969) 938.
- [2] J.J. Matese and R.F. O'Connell, *Astrophys. J.* **160**, (1970) 451.
- [3] J.J. Matese and R.F. O'Connell, *Phys. Rev.* **180**, (1969) 1289.
- [4] For a review of most aspects of primordial nucleosynthesis big bang cosmology, see *The Early Universe*, E.W. Kolb and M.S. Turner, Addison Wesley (1989).
- [5] T. Vachaspati, *Phys. Lett.* **B265** (1991) 258.
- [6] see e.g.: M. Gasperini, M. Giovannini and G. Veneziano, *Phys. Rev. Lett.* **75**, (1995) 3796; A.C. Davis and K. Dimopoulos, CERN-TH/95, DAMPT-95-31, astro-ph/9506132.
- [7] B. Cheng, D.N. Schramm and J.W. Truran, *Phys. Rev.* **D49**, (1994) 5006; *Phys. Lett.* **B316**, (1993) 521.
- [8] L. Kawano, *Let's Go Early Universe: Guide to Primordial Nucleosynthesis Programming*, FERMILAB-PUB-88/34-A. This is a modernized and optimized version of the code written by R.V. Wagoner, *Astrophys. J.* **179**, 343 (1973).
- [9] D. Grasso and H.R. Rubinstein, *Astrop. Phys.* **3**, (1995) 95.
- [10] M. Bander and H.R. Rubinstein, *Phys. Lett.* **B311**, (1993) 187.
- [11] P.J. Kernan, G.D. Starkman and T. Vachaspati, preprint CWRU-P10-95, astro-ph/9509126.
- [12] L. Kawano, *Let's Go Early Universe 2.: Primordial Nucleosynthesis the Computer Way*, FERMILAB-PUB-92/04-A, Jan. 1992.
- [13] Since the updated relic abundances are still subject of discussion (see ref.[21]) we prefer to adopt here a conservative point of view using the abundances reported in: T.P. Walker, G. Steigman, D.N. Schramm, K.A. Olive, and H-S. Kang, *Astrophys. J.* **376**, 51 (1991).
- [14] see Y.B. Zeldovich, A.A. Ruzmakin and D.D. Sokoloff, *Magnetic Fields in Astrophysics*, pag. 293, Gordon and Breach (1983), and references therein.
- [15] K.S. Thorne, *Astrophys. J.* **148**, (1967) 51.
- [16] for a review see P.P. Kronberg, *Rep. Prog. Phys.*, (1994), 323.

- [17] T.W.B. Kibble and A. Vilenkin, *Phys. Rev.* **D52**, (1995) 679.
- [18] G. Baym, D. Bödeker and L. McLerran, TPI-MINN-95-15T, hep-ph/9507429.
- [19] K. Enqvist and P. Olesen, *Phys. Lett.* **B319**, (1993) 178.
- [20] K. Enqvist, A.I. Rez and V.B. Semikoz, *Nucl. Phys.* **B436**, (1995) 49.
- [21] see e.g.: N. Hata et al., *Phys. Rev. Lett.* **75**, (1995) 3977; C.J. Copi, D.N. Schramm and M.S. Turner, *Phys. Rev. Lett.* **75**, (1995) 3981.
- [22] U.H. Danielsson and D. Grasso, *Phys. Rev.* **D52**, (1995) 2533.